Math 215 — Final Exam

December 17, 2019

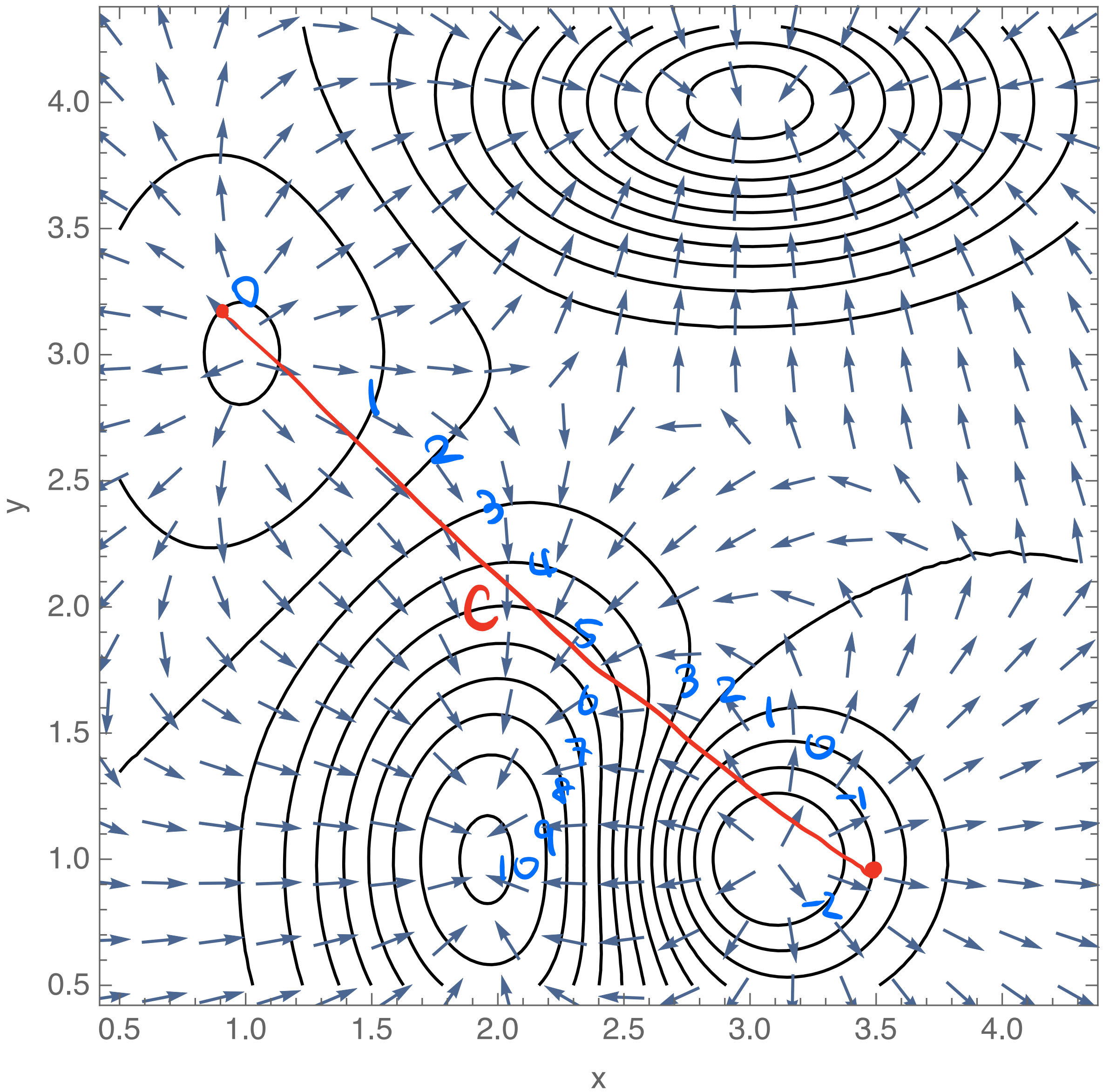
First 3 Letters of Last Name: ☐☐☐ UM Id#: ____________________

Instructor: ____ Section: ____________________

1. **Do not open this exam until you are told to do so.**
2. This exam has 17 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam, other than the formula sheet at the end of the exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. The true or false questions are the only questions that do not require you to show your work. For all other questions show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use no aids (e.g., calculators or notecards) on this exam.
7. **Turn off all cell phones**, remove all headphones, and **place any watch you are using on the desk in front of you**.

Problem	Points	Score
1	10	
2	12	
3	8	
4	10	
5	8	
6	14	
7	10	
8	10	
9	8	
10	10	
Total	100	

1. [10 points] The graph below is a plot of some of the level curves of a function g in a rectangular region $R = [.4, 4.4] \times [.4, 4.4]$. Assume that as we move between adjacent level curves the value of g increases or decreases by exactly one. The arrows point in the direction of ∇g.



a. [4 points] Suppose C is the line segment with initial point at $(.9, 3.2)$ and final point at $(3.5, 1.0)$. What is the approximate value[1] of $\int_C \nabla g \cdot d\mathbf{r}$?

$$\int_C \nabla g \cdot d\vec{r} = g(3.5, 1.0) - g(0.9, 3.2)$$

Fund. Thm.

[1]within .2

Note – Consecutive levels differ by 1

– Arrows $= \nabla g$ point in direction of increase

We can find relative positions of all levels.

Idea Set the level at (0.9, 3.2) to be 0, and find the level at (3.5, 1.0)

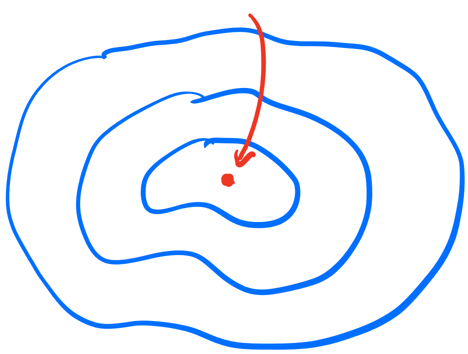
$$\Rightarrow \int_C \nabla g \cdot d\vec{r} = g(3.5, 1.0) - g(0.9, 3.2)$$

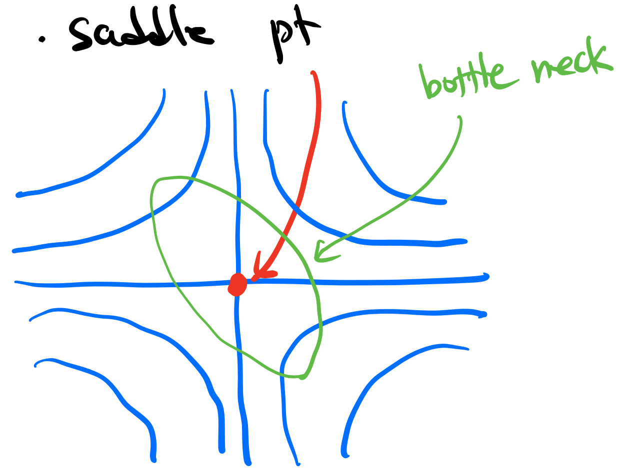
$$= -1 - 0 = \boxed{-1}$$

b. [6 points] Identify the approximate coordinates[2] of the critical points of g in R. For each critical point, indicate if it is a point where the function has a local maximum, a local minimum, or a saddle.

Recall: critical pts typically look as follows:

- loc. max/min
- saddle pt

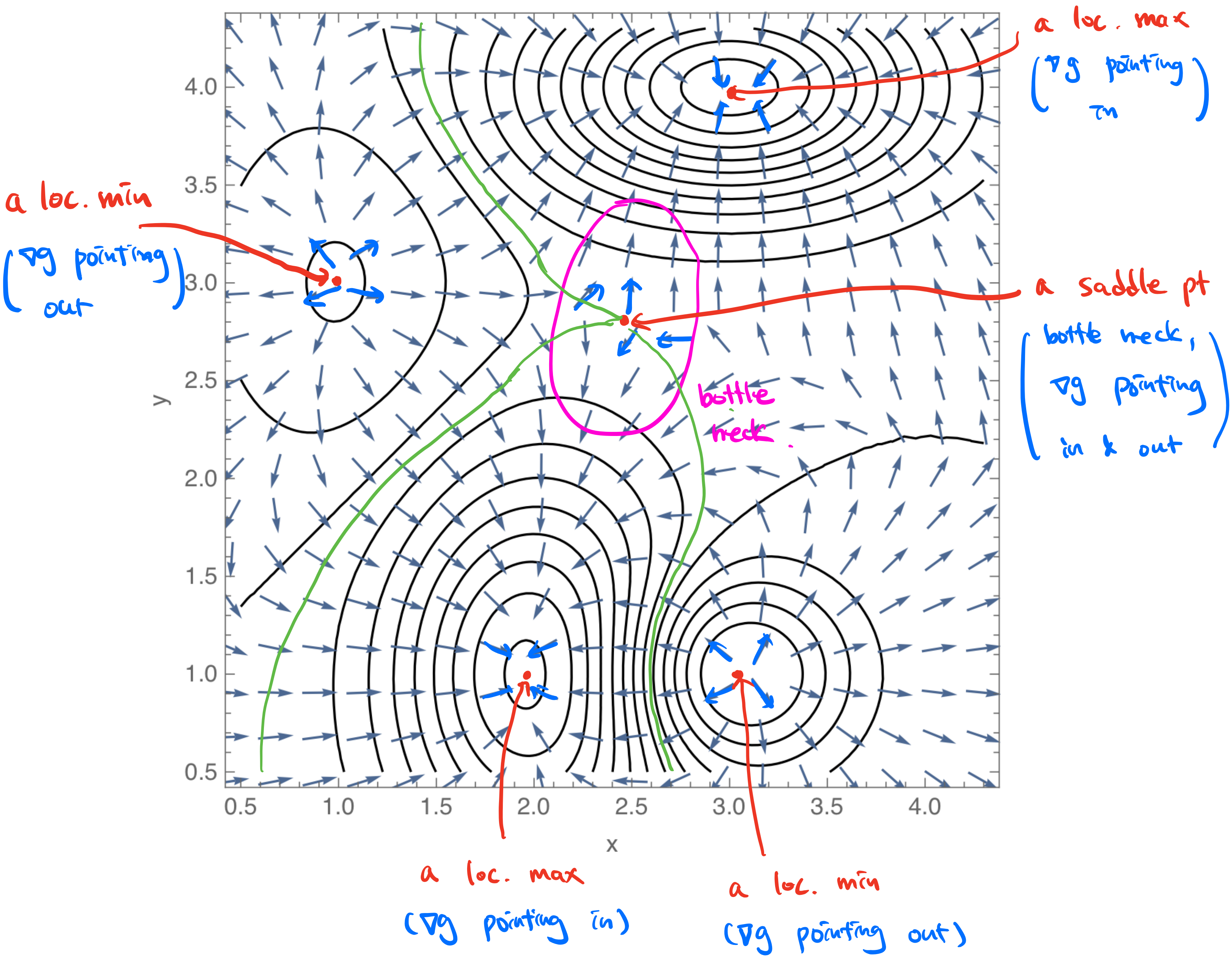




= Center of shrinking curves.

= Intersection of two level curves, or center of a bottle neck

[2]That is, within .1 in each coordinate direction.



$\Rightarrow$

$(1,3)$	:	a loc. min
$(2,1)$	:	a loc. max
$(2.5, 2.8)$	:	a saddle
$(3,1)$	:	a loc. min
$(3,4)$	:	a loc. max

2. [12 points] Some calculations.

a. [4 points] Compute curl($\mathbf{F}$) for $\mathbf{F} = \langle xyz, y\sin(z), y\cos(x)\rangle$.

$P = xyz, \quad Q = y\sin z, \quad R = y\cos x$

$\text{curl}(\vec{F}) = (R_y - Q_z, \; P_z - R_x, \; Q_x - P_y)$

$= (\cos x - y\cos z, \; xy + y\sin x, \; 0 - xz)$

$= \boxed{(\cos x - y\cos z, \; xy + y\sin x, \; -xz)}$

b. [8 points] Suppose $g(x, y, z) = e^{2y}(x^2 + z^3)$. If $x = uvw$, $y = vw/u$ and $z = 2u + vu + w$, then find g_w, the partial derivative of g with respect to w, when $u = -1$, $v = 2$, and $w = 1$.

$$\frac{\partial g}{\partial w} = \frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial w} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial w} + \frac{\partial g}{\partial z}\cdot\frac{\partial z}{\partial w} \qquad (*)$$

chain rule.

$u=-1,\ v=2,\ w=1$

$\Rightarrow x = uvw = -2,\quad y = \frac{vw}{u} = -2,\quad z = 2u + vu + w = -3.$

$$\frac{\partial g}{\partial x} = \frac{\partial}{\partial x} e^{2y}(x^2+z^3) = e^{2y}\cdot 2x = -4e^{-4}.$$

$$\frac{\partial x}{\partial w} = \frac{\partial}{\partial w} uvw = uv = -2.$$

$$\frac{\partial g}{\partial y} = \frac{\partial}{\partial y} e^{2y}(x^2+z^3) = 2e^{2y}(x^2+z^3) = -46e^{-4}.$$

$$\frac{\partial y}{\partial w} = \frac{\partial}{\partial w}\left(\frac{vw}{u}\right) = \frac{v}{u} = -2.$$

$$\frac{\partial g}{\partial z} = \frac{\partial}{\partial z} e^{2y}(x^2+z^3) = e^{2y}\cdot 3z^2 = 27\cdot e^{-4}.$$

$$\frac{\partial z}{\partial w} = \frac{\partial}{\partial w}(2u + vu + w) = 1.$$

$$(*):\ \frac{\partial g}{\partial w} = (-4e^{-4})\cdot(-2) + (-46e^{-4})\cdot(-2) + 27\cdot e^{-4}\cdot 1$$

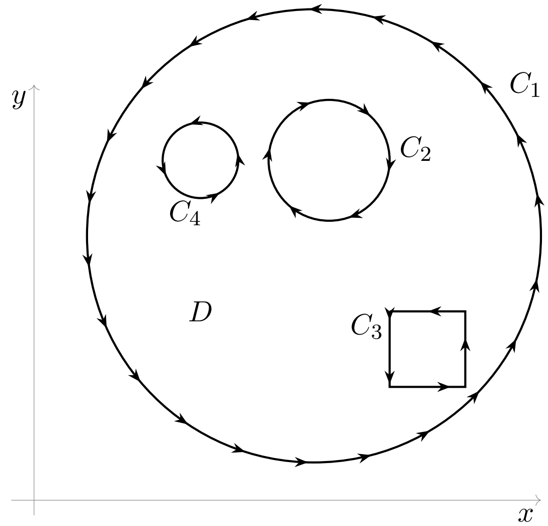
$$= \boxed{127\cdot e^{-4}}$$

3. [8 points] As usual, justify your answers.

a. [4 points] Suppose that D is the bounded region in the plane that has boundary given by the oriented simple closed piece-wise smooth curves C_1, C_2, C_3, and C_4 as in the picture. Suppose $\mathbf{F} = \langle P, Q, 0\rangle : \mathbb{R}^3 \to \mathbb{R}^3$ is a vector field and P and Q have continuous partial derivatives on $\mathbb{R}^2$. If

$$\oint_{C_k} \mathbf{F} \cdot d\mathbf{r} = 2^k,$$

find $\iint_D (Q_x - P_y)\, dA = \iint_D (\nabla \times \mathbf{F}) \cdot \hat{k} dA$.



$\partial D = C_1 \cup C_2 \cup -C_3 \cup -C_4$

$\left(\begin{array}{l} \because C_1, C_2 \text{ are positively oriented.} \\ C_3, C_4 \text{ are negatively oriented} \end{array} \right)$

$$\iint_D (\nabla \times \vec{F}) \cdot \vec{k}\, dA \underset{\text{Green}}{=} \int_{\partial D} \vec{F} \cdot d\vec{r}$$

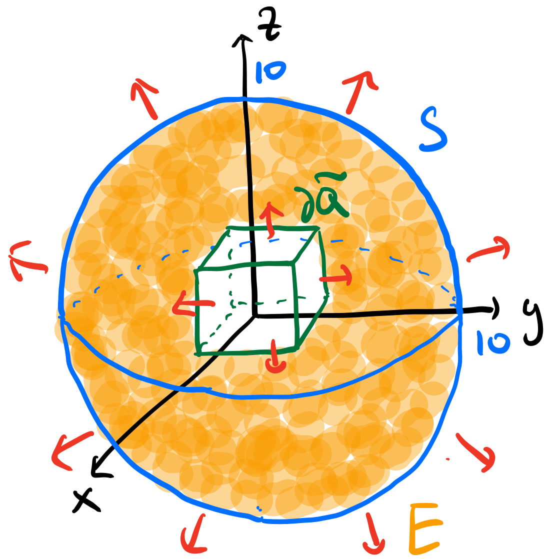
$$= \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} - \int_{C_3} \vec{F} \cdot d\vec{r} - \int_{C_4} \vec{F} \cdot d\vec{r}$$

$$= 2^1 + 2^2 - 2^3 - 2^4$$

$$= \boxed{-18}$$

b. [4 points] Suppose $\mathbf{F}\colon \mathbb{R}^3 \to \mathbb{R}^3$ is a smooth vector field. Let $\tilde{Q}$ be the solid cube centered at the origin with vertices $(\pm 2, \pm 2, \pm 2)$, let $\tilde{S}$ be the sphere centered at the origin of radius 10, and let E be the bounded solid region between $\tilde{S}$ and $\partial\tilde{Q}$. Suppose that the outward flux of $\mathbf{F}$ across $\tilde{S}$ is 5π and the divergence of $\mathbf{F}$ on $\tilde{Q}$ is $\pi/16$. If possible, evaluate the following integral

$$\iiint_E (\nabla \cdot \mathbf{F})\, dV.$$



$\partial E = S \cup -\partial\tilde{Q}$.

($\because$ $\partial\tilde{Q}$ is oriented inward for E)

$$\iiint_E \nabla\cdot\vec{F}\, dV \underset{\substack{\uparrow \\ \text{divergence thm.}}}{=} \iint_{\partial E} \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot d\vec{S} - \iint_{\partial\tilde{Q}} \vec{F}\cdot d\vec{S}$$

$\iint_S \vec{F}\cdot d\vec{S} = 5\pi$ (as given)

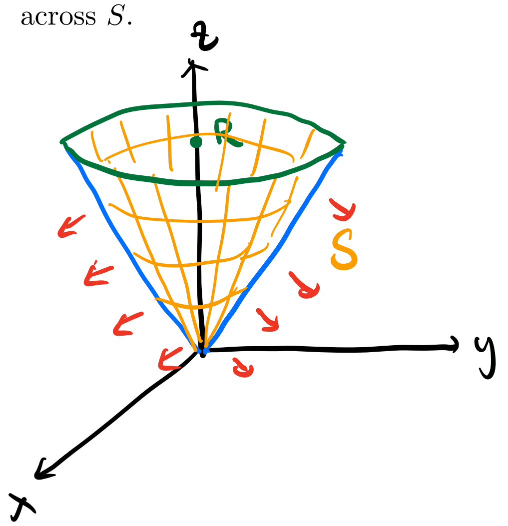
$$\iint_{\partial\tilde{Q}} \vec{F}\cdot d\vec{S} \underset{\substack{\uparrow \\ \text{divergence} \\ \text{thm}}}{=} \iiint_{\tilde{Q}} \underbrace{\nabla\cdot\vec{F}}_{\pi/16}\, dV = \iiint_{\tilde{Q}} \frac{\pi}{16}\, dV$$

$$= \frac{\pi}{16}\iiint_{\tilde{Q}} dV = \frac{\pi}{16}\,\mathrm{Vol}(\tilde{Q}) = \frac{\pi}{16}\cdot 4^3 = 4\pi.$$

$$\Rightarrow \iiint_E \nabla\cdot\vec{F}\, dV = 5\pi - 4\pi = \boxed{\pi}$$

The solution on the archive has an error!

4. [10 points] Suppose R is a positive real number. Let S be the cone given by the equation $z = \sqrt{x^2+y^2}$ with $0 \le z \le R$, oriented downward. Compute the flux of $\mathbf{G} = \langle xz, yz, xy \rangle$ across S.



In cylind. coords:

$$z = \sqrt{x^2+y^2} \rightsquigarrow z = r.$$

$$0 \le z \le R \rightsquigarrow 0 \le r \le R.$$

$\Rightarrow S$ is parametrized by

$$\vec{S}(r,\theta) = (r\cos\theta, r\sin\theta, r)$$

with $0 \le \theta \le 2\pi$, $0 \le r \le R$.

$$\vec{G}(\vec{S}(r,\theta)) = (r\cos\theta \cdot r,\ r\sin\theta \cdot r,\ r\cos\theta \cdot r\sin\theta)$$

$$= (r^2\cos\theta,\ r^2\sin\theta,\ r^2\cos\theta\sin\theta)$$

$$\vec{S}_r = (\cos\theta, \sin\theta, 1), \quad \vec{S}_\theta = (-r\sin\theta, r\cos\theta, 0)$$

$$\vec{S}_r \times \vec{S}_\theta = (-r\cos\theta, -r\sin\theta, \overset{>0}{r})$$: pointing upward.

$$\iint_S \vec{G}\cdot d\vec{S} = \underset{\text{opposite orientation}}{-}\int_0^{2\pi}\int_0^R \vec{G}(\vec{S}(r,\theta))\cdot(\vec{S}_r \times \vec{S}_\theta)\,dr\,d\theta$$

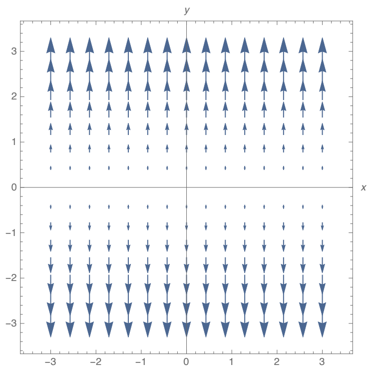
$$= -\int_0^{2\pi}\int_0^R -r^3\cos^2\theta - r^3\sin^2\theta + r^3\cos\theta\sin\theta\,dr\,d\theta$$

$$\int_0^{2\pi}\cos\theta\sin\theta\,d\theta = 0$$

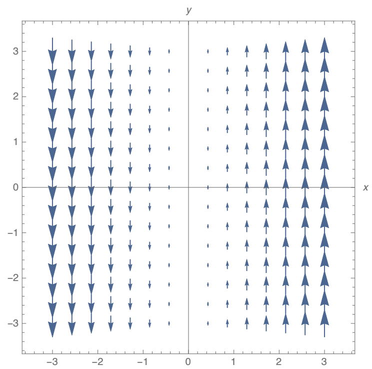
$$= -\int_0^{2\pi}\int_0^R -r^3 + \cancelto{0}{r^3\cos\theta\sin\theta}\,dr\,d\theta = \boxed{\frac{\pi R^4}{2}}$$

5. [8 points] Identify the graph which best represents the vector fields below.

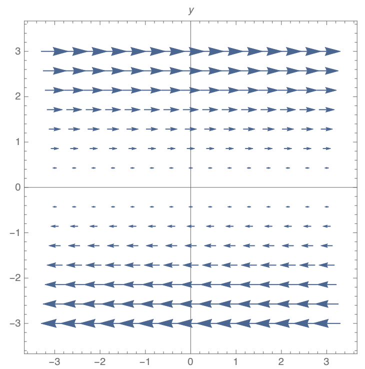
- $\mathbf{F}(x, y) = \langle y, 0\rangle$ ______________________
- $\mathbf{G}(x, y) = \langle x, \cos(y)\rangle$ ______________________
- $\mathbf{H}(x, y) = \langle \sin(y), x\rangle$ ______________________
- $\mathbf{I}(x, y) = \langle x, -y\rangle$ ______________________



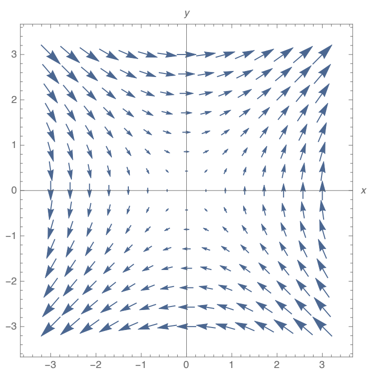
(i)



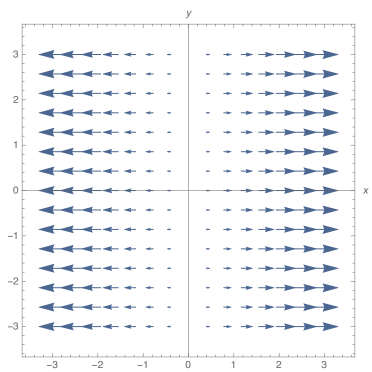
(ii)



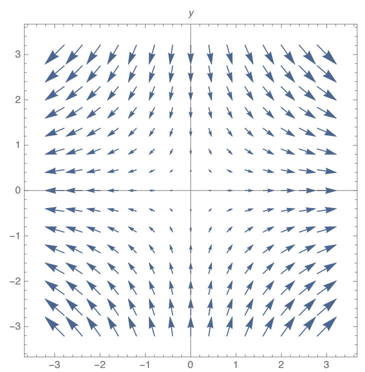
(iii)



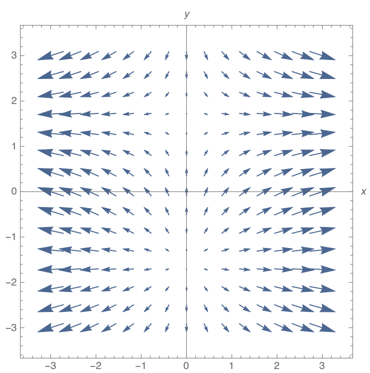
(iv)



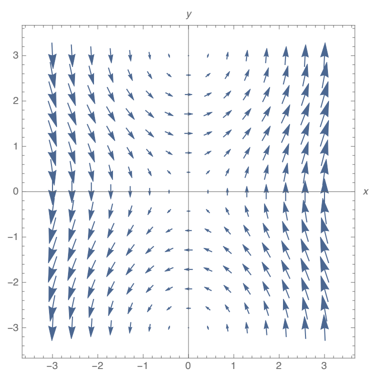
(v)



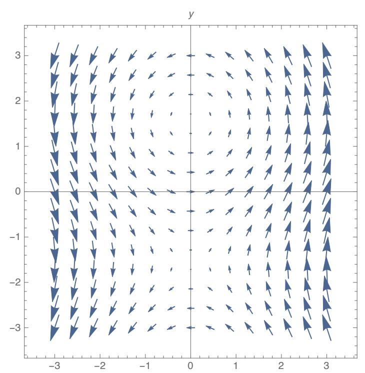
(vi)



(vii)



(viii)



(ix)

Idea: Look at where the vector field is horizontal, vertical or zero.

- $\vec{F}(x,y) = (y,0)$ $\rightsquigarrow$ always horizontal.

  $\vec{F}(x,0) = (0,0)$ $\rightsquigarrow$ zero on x-axis.

  $\Rightarrow$ $\boxed{\text{Match : (iii)}}$

- $\vec{G}(x,y) = (x, \cos(y))$

  $\vec{G}(0,y) = (0, \cos(y))$ $\rightsquigarrow$ vertical on y-axis.

  $\vec{G}(x, \pm\frac{\pi}{2}) = (x, 0)$ $\rightsquigarrow$ horizontal on $y = \pm\frac{\pi}{2}$.

  $\vec{G}(0, \pm\frac{\pi}{2}) = (0,0)$ $\rightsquigarrow$ zero at $(0, \pm\frac{\pi}{2})$

  $\Rightarrow$ $\boxed{\text{Match : (vii)}}$

- $\vec{H}(x,y) = (\sin(y), x)$

  $\vec{H}(0,y) = (\sin y, 0)$ $\rightsquigarrow$ horizontal on y-axis.

  $\vec{H}(x,0) = (0,x)$ $\rightsquigarrow$ vertical on x-axis.

  $\vec{H}(x, \pm\pi) = (0,x)$ $\rightsquigarrow$ vertical on $y = \pm\pi$.

  $\Rightarrow$ $\boxed{\text{Match : (viii)}}$

- $\vec{I}(x,y) = (x, -y)$

  $\vec{I}(0,y) = (0,-y)$ $\rightsquigarrow$ vertical on y-axis

  $\vec{I}(x,0) = (x,0)$ $\rightsquigarrow$ horizontal on x-axis.

  $\Rightarrow$ $\boxed{\text{Match : (vi)}}$

The solution on the archive has an error!

6. [14 points] A failure to justify your answer(s) could result in zero points. The torus T, pictured below, is oriented outward and parameterized by

$$\mathbf{r}(u,v) = \langle (3+\cos(u))\cos(v), (3+\cos(u))\sin(v), \sin(u)\rangle$$

for $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$.

a. [2 points] Does $(0,3,1)$ belong to the surface? If so, what are the values of u and v so that $\mathbf{r}(u,v) = (0,3,1)$?

$$\begin{cases} 0 = (3+\cos(u))\cos(v) \\ 3 = (3+\cos(u))\sin(v) \\ 1 = \sin u \end{cases} \Rightarrow \boxed{u = \frac{\pi}{2}} \Rightarrow \cos(u) = 0.$$

$$3 = 3\sin(v) \Rightarrow \boxed{v = \frac{\pi}{2}}$$

b. [8 points] Find an equation for the tangent plane to T at $\mathbf{r}(\pi/3, \pi/6) = (7\sqrt{3}/4, 7/4, \sqrt{3}/2)$.

$$\vec{r_u} = (-\sin(u)\cos(v), -\sin(u)\sin(v), \cos(u))$$

$$\vec{r_v} = (-(3+\cos(u))\sin(v), (3+\cos(u))\cos(v), 0)$$

$$\begin{cases} u = \frac{\pi}{3} \Rightarrow \sin(u) = \frac{\sqrt{3}}{2}, \quad \cos(u) = \frac{1}{2}. \\ v = \frac{\pi}{6} \Rightarrow \sin(v) = \frac{1}{2}, \quad \cos(v) = \frac{\sqrt{3}}{2} \end{cases}$$

$$\rightsquigarrow \vec{r_u} = \left(-\frac{3}{4}, -\frac{\sqrt{3}}{4}, \frac{1}{2}\right), \quad \vec{r_v} = \left(-\frac{7}{4}, \frac{7\sqrt{3}}{4}, 0\right)$$

$$\vec{r_u} \times \vec{r_v} = \left(-\frac{7\sqrt{3}}{8}, -\frac{7}{8}, -\frac{14\sqrt{3}}{8}\right) : \text{normal vector.}$$

$$\rightsquigarrow \boxed{-\frac{7\sqrt{3}}{8}\left(x - \frac{7\sqrt{3}}{4}\right) - \frac{7}{8}\left(y - \frac{7}{4}\right) - \frac{14}{8}\sqrt{3}\left(z - \frac{\sqrt{3}}{2}\right) = 0}$$

* You can write

$$\vec{r_u} \times \vec{r_v} = -\frac{7}{8}(\sqrt{3}, 1, 2\sqrt{3})$$

and take $\vec{n} = (\sqrt{3}, 1, 2\sqrt{3})$ as a normal vector

$$\rightsquigarrow \sqrt{3}\left(x - \frac{7\sqrt{3}}{4}\right) + 1\cdot\left(y - \frac{7}{4}\right) + 2\sqrt{3}\left(z - \frac{\sqrt{3}}{2}\right) = 0$$

c. [4 points] Compute

$$\iint_T (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

for $\mathbf{F} = \langle xyz, y\sin(z), y\cos(x)\rangle$.

* This is a special case of HW 10 #113 (b).

Sol 1

$$\iint_T (\nabla \times \vec{F}) \cdot d\vec{S} \underset{\text{Stoke}}{=} \int_{\partial T} \vec{F} \cdot d\vec{r} \underset{\partial T \text{ is empty}}{=} \boxed{0}$$

Sol 2 Take E to be the solid bounded by T.

$$\iint_T (\nabla \times \vec{F}) \cdot d\vec{S} \underset{T = \partial E}{=} \iint_{\partial E} (\nabla \times \vec{F}) \cdot d\vec{S}$$

$$\underset{\text{divergence thm}}{=} \iiint_E \underbrace{\nabla \cdot (\nabla \times \vec{F})}_{0} \, dV = \boxed{0}$$

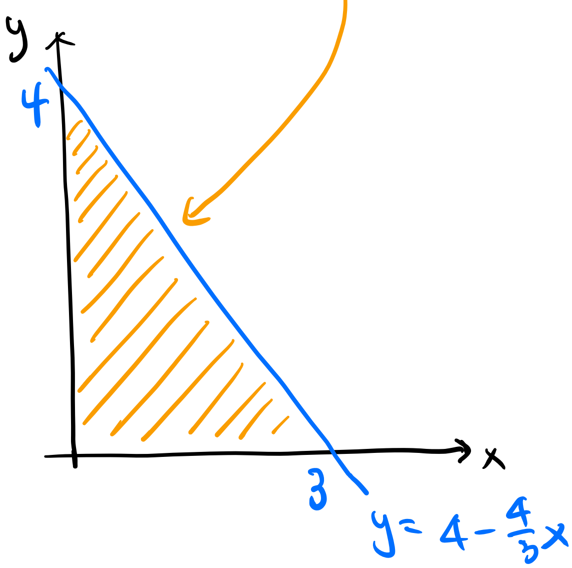
div(curl($\vec{F}$)) = 0 for all smooth vector fields.

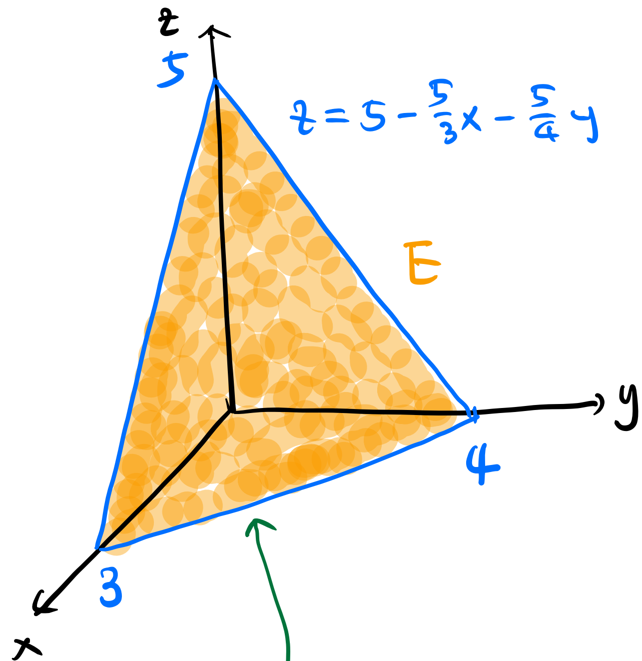
7. [10 points] Let $\alpha\colon \mathbb{R}^3 \to \mathbb{R}$ be a continuous function. Consider the integral

$$\int_0^3 \int_0^{4-(4/3)x} \int_0^{5-(5/3)x-(5/4)y} \alpha(x,y,z)\,dz\,dy\,dx.$$

a. [4 points] Sketch the region of integration.

$0 \le x \le 3,\ \underline{0 \le y \le 4 - \frac{4}{3}x},\ 0 \le z \le 5 - \frac{5}{3}x - \frac{5}{4}y.$





$* z = 5 - \frac{5}{3}x - \frac{5}{4}y$ is a plane with

x-intercept 3, y-intercept 4, z-intercept 5.

b. [6 points] If we change the order of integration to

$$\int_a^b \int_c^d \int_e^f \alpha(x,y,z)\,dx\,dz\,dy,$$

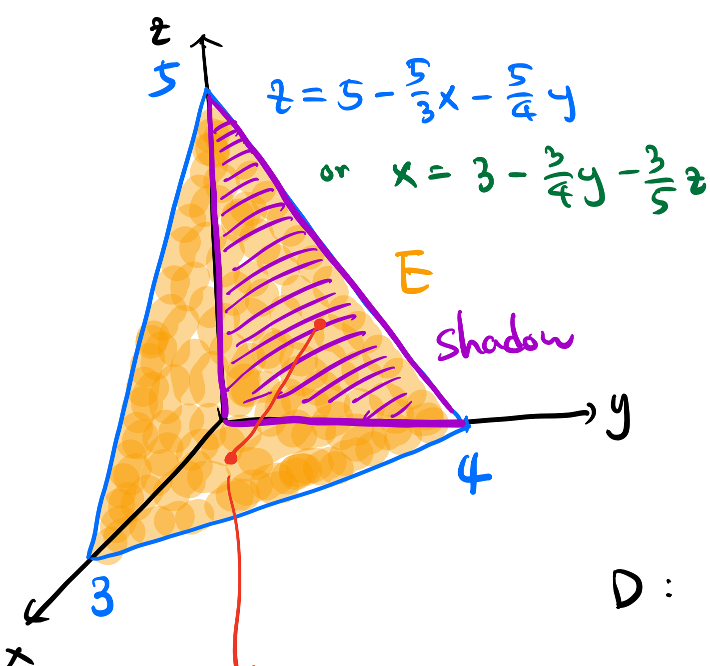
then

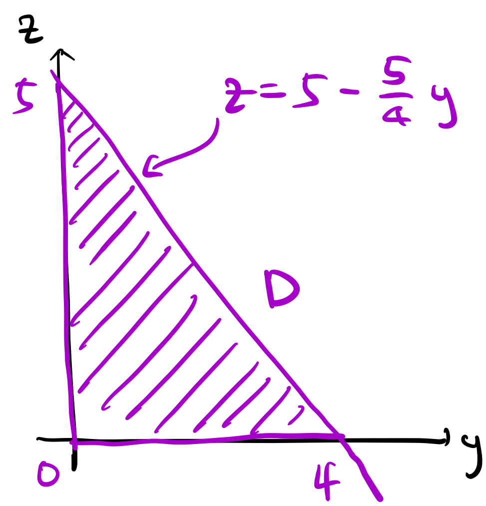
- a is 0
- b is 4
- c is 0
- d is $5-\frac{5}{4}y$
- e is 0
- f is $3-\frac{3}{4}y-\frac{3}{5}z$

<u>Sol 1</u> (Projection method)

dx on the innermost integral

⇝ look at shadow on yz-plane.





D : $0 \le y \le 4$, $0 \le z \le 5-\frac{5}{4}y$.

For each point on D : $0 \le x \le 3-\frac{3}{4}y-\frac{3}{5}z$.

$$\Rightarrow \int_0^4 \int_0^{5-\frac{5}{4}y} \int_0^{3-\frac{3}{4}y-\frac{3}{5}z} \alpha(x,y,z)\,dx\,dz\,dy.$$

Sol 2 (Cross section method)

$0 \le x \le 3, \quad 0 \le y \le 4 - \frac{4}{3}x, \quad 0 \le z \le 5 - \frac{5}{3}x - \frac{5}{4}y.$

Bounds for y: $0 \le y \le 4$ from picture

(or $0 \le y \le 4 - \frac{3}{4}x \le 4$)

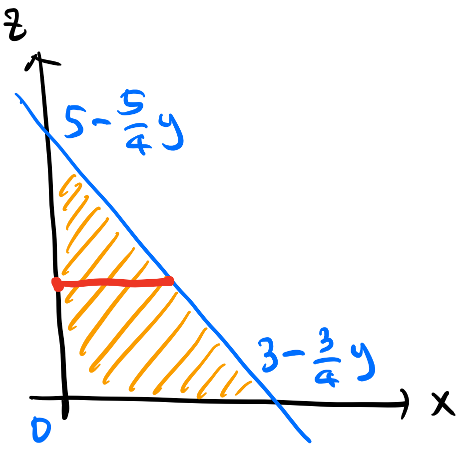
$0 \le x$

Cross section for x and z:

$0 \le x \le 3, \quad x \le 3 - \frac{3}{4}y, \quad 0 \le z \le 5 - \frac{5}{3}x - \frac{5}{4}y$

$y \le 4 - \frac{4}{3}x$

$\Rightarrow \quad 0 \le x \le 3 - \frac{3}{4}y, \quad 0 \le z \le 5 - \frac{5}{3}x - \frac{5}{4}y.$



$z = 5 - \frac{5}{3}x - \frac{5}{4}y.$

or $x = 3 - \frac{3}{4}y - \frac{3}{5}z$

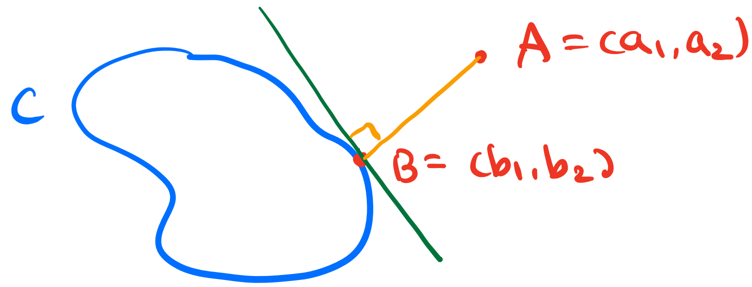
D: $0 \le z \le 5 - \frac{5}{4}y, \quad 0 \le x \le 3 - \frac{3}{4}y - \frac{3}{5}z$

$$\Rightarrow \int_0^4 \int_0^{5 - \frac{4}{5}y} \int_0^{3 - \frac{3}{4}y - \frac{3}{5}z} \delta(x,y,z)\, dx\, dz\, dy.$$

*This problem is essentially HW 6J #65.

8. [10 points] Suppose C is a smooth simple closed curve in the plane and $A = (a_1, a_2)$ is a point in the plane outside of the region bounded by the curve C. Let $B = (b_1, b_2)$ be a point on the curve closest to A.

   a. [2 points] Draw a picture that illustrates the situation.



   b. [8 points] Use the method of Lagrange multipliers to explain why the tangent line to C at the point B is perpendicular to the line that contains A and B.

Describe C by $g(x,y)=0$.

Distance from A is $\sqrt{(x-a_1)^2 + (y-a_2)^2}$

$B=(b_1,b_2)$ minimizes $f(x,y) = (x-a_1)^2 + (y-a_2)^2$

subject to $g(x,y)=0$

Lagrange multipliers : $\nabla f(b_1,b_2) = \lambda \nabla g(b_1,b_2)$ (*)

$\nabla f = \left(2(x-a_1), 2(y-a_2)\right)$

$\nabla f(b_1,b_2) = \left(2(b_1-a_1), (b_2-a_2)\right) = 2\overrightarrow{AB}$.

(*) $\Rightarrow$ $\overrightarrow{AB}$ is parallel to $\nabla g(b_1,b_2)$

Also, $\nabla g(b_1,b_2)$ is perpendicular to the tangent line at $B=(b_1,b_2)$

$\Rightarrow$ $\overrightarrow{AB}$ is perpendicular to the tangent line.

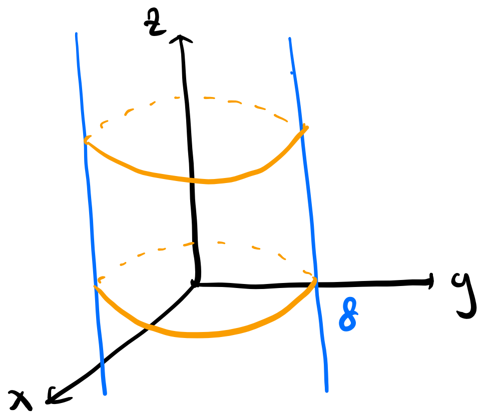
9. [8 points] The following equations are expressed in spherical coordinates. Sketch the solid or surface described by each equation.

a. [4 points] $8 - \rho \sin \phi = 0$

In cylindrical coordinates : $r = \rho \sin \varphi$

$\Rightarrow 8 - \rho \sin \varphi = 0 \rightsquigarrow 8 - r = 0 \rightsquigarrow r = 8$.

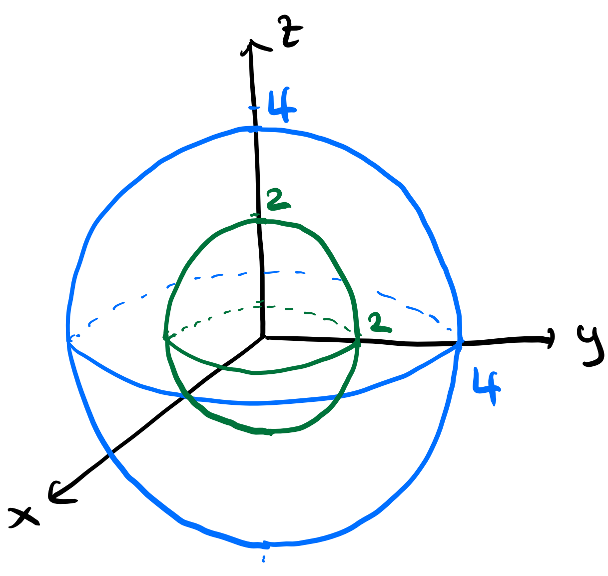
$\rightsquigarrow$ a cylinder of radius 8 along z-axis.



b. [4 points] $\rho^2 = 6\rho - 8$

$\rho^2 - 6\rho + 8 = 0 \rightsquigarrow (\rho - 2)(\rho - 4) = 0 \rightsquigarrow \rho = 2$ or 4

$\rightsquigarrow$ spheres of radii 2 and 4 centered at the origin.



10. [10 points] Indicate if each of the following is true or false by circling the correct answer. No partial credit will be given.

a. [2 points] Consider the function $y = 4\cos(t)$ where $0 \leq t \leq 5$ is measured in seconds. The following commands are entered in MatLab:

```
>> t = 0:.01:5;
>> y = 4*cos(t);
```

If you type $y(1)$, then MatLab will return 4.

$y(1)$ returns the first entry.

True False

$\Rightarrow y(1) = 4\cos(0) = 4.$

b. [2 points] If $\mathbf{G}\colon \mathbb{R}^3 \to \mathbb{R}^3$ is a smooth vector field, then $\nabla(\operatorname{div}\mathbf{G}) = \mathbf{0}$.

Take $\vec{G} = (x^2, 0, 0)$

True False

$\Rightarrow \operatorname{div}(\vec{G}) = 2x, \quad \nabla(\operatorname{div}(\vec{G})) = (2, 0, 0) \neq \vec{0}.$

c. [2 points] The result of integrating a vector field over a surface is a vector.

A surface integral is always a scalar.

True False

d. [2 points] The integral of a vector field over a smooth closed curve is always zero.

This is not necessarily true if the vector field is not conservative

True False

e. [2 points] If $x = uv$ and $y = u^2 - v^2$, then the Jacobian of the transformation is $2u^2 + 2v^2$.

$$\left| \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} \right| = \left| \det \begin{bmatrix} v & u \\ 2u & -2v \end{bmatrix} \right|$$

True False

Don't forget the abs. value!

$$= |-2v^2 - 2u^2| = 2v^2 + 2u^2.$$

This sheet will not be graded. Do not turn it in.

- $\sin^2(x) + \cos^2(x) = 1$, $\cos(2x) = \cos^2(x) - \sin^2(x)$, $\sin(2x) = 2\sin(x)\cos(x)$
- $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$, $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$
- $\cos(\pi/3) = 1/2$, $\sin(\pi/3) = \sqrt{3}/2$, $\cos(\pi/4) = \sqrt{2}/2$, $\sin(\pi/4) = \sqrt{2}/2$, $\cos(\pi/6) = \sqrt{3}/2$, $\sin(\pi/6) = 1/2$, $\cos(0) = 1$, $\sin(0) = 0$.
- $\frac{d}{dx}\sin(x) = \cos(x)$, $\quad \frac{d}{dx}\cos(x) = -\sin(x)$.
- Volume of the parallelepiped determined by the vectors $\mathbf{v_1} = \langle a,b,c\rangle$, $\mathbf{v_2} = \langle d,e,f\rangle$, and $\mathbf{v_3} = \langle g,h,i\rangle$ is $|\mathbf{v_1}\cdot(\mathbf{v_2}\times\mathbf{v_3})| =$ absolute value of $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$
- Distance from a point (a,b,c) to a plane $Ax + By + Cz + D = 0$ is $\frac{|Aa+Bb+Cc+D|}{\sqrt{A^2+B^2+C^2}}$.
- The circumference of a circle of radius a is $2\pi a$.
- The area of a disk of radius a is πa^2.
- The volume of a right circular cylinder of radius a and height h is $\pi a^2 h$.
- The curvature of the curve given by the parametric equation $\mathbf{r}(t)$ is $\kappa(t) = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$.
- $\int \sin^2(u)\,du = \frac{u}{2} - \frac{\sin(2u)}{4} + C$ $\qquad \int \cos^2(u)\,du = \frac{u}{2} + \frac{\sin(2u)}{4} + C$
- $\int \ln(u)\,du = u\ln(u) - u + C$
- The volume of a right circular cylinder of radius a and height h is $\pi a^2 h$.
- The volume of a sphere of radius a is $\frac{4\pi a^3}{3}$.
- The surface area of a sphere of radius a is $4\pi a^2$.
- The volume of a cone with base radius a and height b is $\frac{1}{3}\pi a^2 b$.
- Polar coordinates $x = r\cos(\theta)$, $y = r\sin(\theta)$.
- Cylindrical coordinates $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = z$.
- Spherical coordinates $x = \rho\cos(\theta)\sin(\phi)$, $y = \rho\sin(\theta)\sin(\phi)$, $z = \rho\cos(\phi)$.
- Green's Theorem:
$$\oint_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$
- Stokes' Theorem:
$$\oint_{\partial S} \vec{F}\cdot d\vec{r} = \iint_S \mathrm{curl}\vec{F}\cdot d\vec{S}$$
- Divergence Theorem:
$$\iint_{\partial E} \vec{F}\cdot d\vec{S} = \iiint_E (\mathrm{div}\vec{F})\,dV.$$